**Assignment 3 – Part 1**

**Set 4.6 - 12, 16, 28 (Prove by Contradiction), Set 4.7 - 8, 16.c (Prove by Contradiction)**

**Set 4.6 - 12, 16, 28 (Prove by Contradiction)**

12.) If a and b are rational numbers, b0, and r is an irrational number, then a + br is irrational.

**Proof (by contradiction):**

**Assume for the sake of contradiction *a* and *b* are rational numbers, , *r* is an irrational number, and is rational. [We must deduce a contradiction.] By definition of rational, , , and for some integers *c*, *d, e, f, g,* and *h* with . By substitution and algebra,**

**Now and are both integers since products and differences of integers are integers, by the zero-product property. Hence, *r* is the quotient of the two integers and with . Thus, by definition of rational, *r* is rational, which contradicts the supposition that *r* is irrational.** **[Hence the supposition is false and the theorem is true.]**

16.) For all odd integers a, b, and c, if z is a solution of then z is irrational. (In the proof, use the properties of even and odd integers that are listed in Example 4.2.3.p195)

**Proof (by contradiction):**

**Assume for the sake of contradiction a, b, and c are odd numbers, z is a solution of , and z is rational. [We must deduce a contradiction.] By definition of rational, for some integers e and f with . We may assume that e and f have no common factor because if they did, . Because e and f have no common factor, they are not both even. By substitution,**

**Multiply whole equation by**

**Case 1 (assuming e is even and f is odd)**

**Assume e if even and f is odd. The expressions and will be even because any product of an even and odd integer is even. will be odd because the product of any two odd integers is odd. Adding will result in an odd number because the sum of any odd and even integer is odd. This is a contradiction because 0 is not odd.**

**Case 2 (assuming f is even and e is odd)**

**Assume e if odd and f is even. The expressions will be even because any product of an even and odd integer is even. will be odd because the product of any two odd integers is odd. Adding will result in an odd number because the sum of any odd and even integer is odd. This is a contradiction because 0 is not odd.**

**Case 3 (assuming e is odd and f is odd)  
Assume e and f are odd. will be odd because any product of odd integers is odd. The sum of any 2 odd integers is even, and the sum of an even integer with an odd integer is odd. Thus, adding will result in an odd number. Again, this is a contradiction because 0 is not odd.**

**This leaves the case in which e and f are both even, which was supposed to be impossible. [Hence the supposition is false and the theorem is true.]**

28.) For all integers m and n, if mn is even then m is even or n is even.

**Proof (by contradiction):**

**Assume for the sake of contradiction, is even, is odd, and is odd. By definition in Example 4.2.3, an odd integer times another odd integer is odd. Therefore, must be odd, which contradicts our supposition that is even. [Hence the supposition is false and the theorem is true.]**

**Set 4.7 - 8, 16.c (Prove by Contradiction) p.212**

8.) The difference of any two irrational numbers is irrational. **FALSE.**

**Proof (by counterexample):**

**0 is not rational because it can be written in a rational form**

16.c.) Prove that √3 is irrational.

**Proof (by contradiction):**

**Assume for the sake of contradiction is rational. Then there are integers *m* and *n* with no common factors and such that**

**Thus is divisible by 3, and so, by 16.b., is also divisible by 3. By definition of divisibility, for some integer , and so**

**Hence is divisible by 3, and so, by 16.b., *n* is also divisible by 3. Consequently, *m* and *n* are both divisible by 3, which contradicts our assumption that *m* and *n* have no common factor. [Hence the supposition is false and the theorem is true.]**